KINETICS OF HEAT TRANSFER DURING THE DESICCATION OF MOIST MATERIALS

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A relation is established between the Rebinder number and the Nusselt number for the slowing-down stage of the desiccation process, also between the Nusselt number and the dimensionless desiccation rate during the first and the second stage of the process.

As is well known, the desiccation rate of moist materials depends on both the internal and the external heat and mass transfer. The heat and the mass transfer are interrelated processes. In view of this, in any analysis they must be considered together.

In order to describe the kinetics of desiccation, one must know the trend of the mean-integral characteristics of the moist material ($\bar{u}$, $\bar{t}_m$). It would appear that the laws of external heat and mass transfer could be used here, j and q for the slowing-down stage of desiccation cannot be calculated by Dalton's equation and by Newton's equation, however, because the coefficients of heat and mass transfer vary with time. Desiccation is a transient process.

In order to use the Newton formula, various authors have introduced corrections to it. I. M. Fedorov [1] and P. D. Lebedev [2] have established experimentally that the heat transfer coefficient is constant only during the constant-rate stage of desiccation, while it decreases with decreasing moisture content during the slowing-down stage of desiccation and gradually approaches the heat transfer coefficient for the absolutely dry material.

P. D. Lebedev proposed a correction to Newton's formula and assumed that the heat transfer coefficient during the slowing-down stage of desiccation varies as a function of the moisture W according to the empirical equation

$$\frac{\alpha_W}{\alpha_{cr}} = \left(\frac{W}{W_{cr}}\right)^n. \tag{1}$$

An evaluation of test data on the desiccation of various materials has yielded the following formula for the Nusselt number of heat transfer [2]:

$$\mathrm{Nu} = A\,\mathrm{Re}^p \left(\frac{T_m}{T_w}\right)^m \left(\frac{T_r}{T_m}\right)^k \left(\frac{W}{W_{cr}}\right)^n. \tag{2}$$

Subsequent studies have shown that the exponent of the temperature factor in Eq. (2) for convective desiccation is m = 2 and does not depend on the kind of the dried material.

From the values of the coefficients in Eq. (2) compiled in the technical literature [3, 4] for several materials we have established the following correlation between the fundamental equation of desiccation kinetics

$$q_s(\tau) = \rho_0 R_V r \frac{d\bar{u}}{d\tau} (1 + \mathrm{Rb}) \tag{3}$$

and the empirical Nusselt equation (2) of heat transfer.

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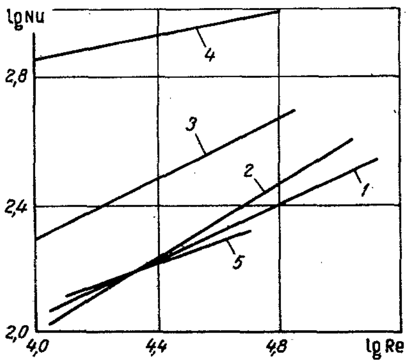

Fig. 1

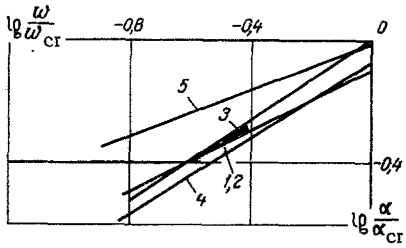

Fig. 2

Fig. 1. Nusselt number as a function of the Reynolds number, during the first stage of desiccation, for: felt (1), asbestos (2), peat slab (3), carrots (4), food antibiotic (Actinomices griseus) (5).

Fig. 2. Ratio α/α_{cr} as a function of the ratio W/W_{cr} for: felt (1), asbestos (2), peat slab (3), carrots (4), food antibiotic (Actinomices griseus) (5).

The desiccation rate during the first stage can be determined from the known thermal flux density according to the formula

$$N = \frac{q_{fs}}{r\rho_0 R_V}. \tag{4}$$

The thermal flux density or the heat transfer rate during the first stage can be determined with the aid of Newton's equation, and (4) will then become

$$N = \frac{q_{fs}}{r\rho_0 R_V} = \frac{\alpha_{cr}(T_m - T_w)}{r\rho_0 R_V}. \tag{5}$$

During the second stage the desiccation rate is approximately

$$\frac{d\bar{u}}{d\tau} = -\varkappa N(\bar{u} - u_e). \tag{6}$$

The relative desiccation factor, which depends on the properties of the material and on the initial moisture content, is equal to

$$\varkappa = \frac{1}{\bar{u}_{rc} - u_e}. \tag{7}$$

The thermal flux density during the slowing-down stage of desiccation is, according to Newton's equation of convective heat transfer,

$$q_s = \bar{\alpha}(T_m - T_{sur}). \tag{8}$$

Inserting (6) and (7) into Eq. (3) yields

$$\bar{\alpha}(T_m - T_{sur}) = \rho_0 R_V r \varkappa N(\bar{u} - u_e)(1 + \mathrm{Rb}). \tag{9}$$

Substituting expression (5) for N, we have

$$\bar{\alpha}(T_m - T_{sur}) = \varkappa(\bar{u} - u_e)\alpha_{cr}(T_m - T_w)(1 + \mathrm{Rb})$$

or

$$\frac{\bar{\alpha}(T_m - T_{sur})}{\alpha_{cr}(T_m - T_w)} = \varkappa(\bar{u} - u_p)(1 + \mathrm{Rb}), \tag{10}$$

but, according to relation (1),

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_{cr}} = \frac{\bar{\alpha}}{\alpha_{cr}} = \frac{\bar{u}}{u_{cr}}, \tag{11}$$

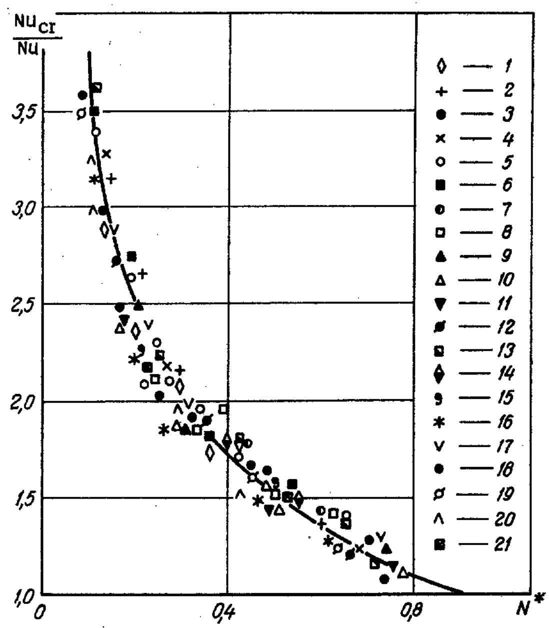

Fig. 3. Ratio Nu_{cr}/Nu as a function of N*, for: wood (pine) at T_m = 120°C with ϑ = 10 m/sec and δ = 10 mm (1), 20 mm (2), for felt at T_m = 120°C with φ = 5% and δ = 8 mm, ϑ = 3 m/sec (3), 5 m/sec (4), 10 m/sec (5), 15 m/sec (6), T_m = 150°C and ϑ = 5 m/sec (7), for asbestos at T_m = 120°C with φ = 5% and δ = 6 mm, ϑ = 3 m/sec (8), 5 m/sec (9), 10 m/sec (10), 15 m/sec (11), T_m = 90°C with ϑ = 5 m/sec (12), for ceramics at T_m = 120°C with φ = 5% and δ = 5 mm, ϑ = 3 m/sec (13), 5 m/sec (14), 10 m/sec (15), for carrots at T_m = 80°C with φ = 5% and ϑ = 3 m/sec (16), 5 m/sec (17), 10 m/sec (18), for clay at T_m = 120°C with φ = 5% and ϑ = 5 m/sec, δ = 10 mm (19), 20 mm (20), 50 mm (21).

and then

$$(1+\mathrm{Rb})\,\varkappa(\bar{u}-u_e)=\left(\frac{T_m-T_{sur}}{T_m-T_w}\right)\left(\frac{\bar{u}}{\bar{u}_{cr}}\right)^n \qquad (12)$$

or

$$(1+\mathrm{Rb})=\left(\frac{\bar{u}_{rc}-u_e}{\bar{u}-u_e}\right)\left(\frac{T_m-T_{sur}}{T_m-T_w}\right)\left(\frac{\bar{u}}{\bar{u}_{cr}}\right). \qquad (13)$$

If the Rebinder number has been based on the specific heat of the moist material, while it should be calculated with only the specific heat of the dry material, then both values are related as follows:

$$\mathrm{Rb}=\mathrm{Rb}_0\left(1+\frac{c_M\bar{u}}{c_0}\right), \qquad (14)$$

and then

$$1+\mathrm{Rb}=\mathrm{Rb}_0\left(1+\frac{c_M\bar{u}}{c_0}\right)+1=\left(\frac{\bar{u}_{rc}-u_e}{\bar{u}-u_e}\right)\left(\frac{T_m-T_{sur}}{T_m-T_w}\right)\left(\frac{\bar{u}}{\bar{u}_{cr}}\right)^n. \qquad (15)$$

Replacing $(\bar{u}/\bar{u}_{cr})^n$ by Nu/Nu_{cr}, therefore, will yield a relation between the Rebinder number and the Nusselt number for the slowing-down second stage of desiccation:

$$\mathrm{Nu}=\mathrm{Nu}_{cr}\left(\frac{T_m-T_w}{T_m-T_{sur}}\right)\left(\frac{\bar{u}-u_e}{\bar{u}_{rc}-u_e}\right)(1+\mathrm{Rb}) \qquad (16)$$

and, with the use of the relation

$$\mathrm{Nu}=\frac{\bar{\alpha}l}{\lambda_A}, \qquad (17)$$

we will have

$$\mathrm{Nu}\,\frac{\lambda_A}{l}\,(T_m-T_{sur})=\rho_0 R_V r\,\frac{d\bar{u}}{d\tau}\,(1+\mathrm{Rb}) \qquad (18)$$

or

$$\mathrm{Nu}=\rho_0 R_V r l\,\frac{d\bar{u}}{d\tau}\cdot\frac{1+\mathrm{Rb}}{\lambda_A\,(T_m-T_{sur})}. \qquad (19)$$

Since so far there are no reliable formulas available for calculating the time that particles of a material remain on the surface of the material in spray desiccators or in gaseous suspension devices, hence one often uses the concept of a heat transfer coefficient referred to a unit volume of the apparatus:

$$\alpha_V=\alpha\,\frac{F_n}{V_d}=\frac{6\alpha}{d_\tau}\,\beta. \qquad (20)$$

The relation between the volume Nusselt number Nu_V and the Rebinder number is of the same type (see expression (19)).

Our experiments have yielded a relation of the type (2) for felt, asbestos, peat slab, carrots, and food antibiotic (Actinomices griseus) (Figs. 1 and 2). In Table 1 are given the values of the coefficients in Eq. (2) as well as of the Nusselt number according to (2) and (16).

According to the table, the values of the Nusselt number calculated by formulas (2) and (16) agree rather closely. The Nusselt number for carrots was calculated by formulas (2) and (19).

An analysis of test data indicates the feasibility of establishing, through the desiccation rate, a relation between the constant Nusselt number for the first stage of desiccation and the variable Nusselt number for the second stage of the process.

TABLE 1

Material	A	k	p	n	Nu (2)	Nu (16)
Felt	0,435	2	0,5	0,5	28	30
Asbestos	0,75	2	0,5	0,5	83	77
Peat slab	1,1	2	0,5	0,65	143	154
Carrots	4,0	2	0,2	0,65	552	560 (19)
Food antibiotic (Actinomices griseus)	5,5	2	0,031	0,4	142	151

Introducing the parameters

$$\Delta T^* = \frac{T_m - T_{sur}}{T_m - T_w}; \quad N^* = \frac{d\bar{u}/d\tau}{N} \tag{21}$$

and inserting them into expression (10) will yield

$$\Delta T^* = N^* (1 + \mathrm{Rb}) \frac{\alpha_{cr}}{\alpha} \tag{22}$$

or

$$\Delta T^* = N^* (1 + \mathrm{Rb}) \frac{\mathrm{Nu}_{cr}}{\mathrm{Nu}}. \tag{23}$$

The Rebinder number for materials such a cloth varies from 0.064 to 0.04 over the $\bar{u}$ = 0.22–0.30 kg/kg range of moisture content, for ceramic tiles δ = 6.5 mm thick it varies from 0.03 to 0.30 over the $\bar{u}$ = 0.1–0.03 kg/kg range of moisture content, and for asbestos it varies from 0.30 to 0.02 over the $\bar{u}$ = 0.02–0.18 kg/kg range of moisture content. Thus, disregarding the small Reynolds number here, one can write

$$\Delta T^* = N^* \frac{\mathrm{Nu}_{cr}}{\mathrm{Nu}}. \tag{24}$$

Since

$$\frac{\mathrm{Nu}_{cr}}{\mathrm{Nu}} = f(N^*),$$

hence we can plot the values of $\mathrm{Nu}_{cr}/\mathrm{Nu}$ along the ordinates against the values of N^* along the abscissas. According to Fig. 3, all test points within the desiccation range for such materials as wood, felt, asbestos, ceramics, and clay fit on a single curve without deviating more than 10–15%. The $\log \mathrm{Nu}_{cr}/\mathrm{Nu}$ versus $\log N^*$ curve yields

$$\frac{\mathrm{Nu}_{cr}}{\mathrm{Nu}} = N^{*-0.57} \tag{25}$$

or

$$\mathrm{Nu} = \mathrm{Nu}_{cr} N^{*0.57}. \tag{26}$$

Thus, formula (26) is a universal one for capillary-porous colloidal materials and yields the Nusselt number for the second stage of desiccation with an error not larger than 10–15% when the Biot number $\mathrm{Bi}_q \le 1$.

With the Rebinder number taken into account, formula (25) becomes

$$(1 + \mathrm{Rb}) \frac{\mathrm{Nu}_{cr}}{\mathrm{Nu}} = N^{*-0.57}. \tag{27}$$

Inserting (27) into (23) yields

$$\Delta T^* = N^{*0.43}$$

and

$$T_{sur} = T_m - N^{*0.43} (T_m - T_w). \tag{28}$$

Equation (29) makes it possible, with the desiccation rate known, to determine the temperature at the material surface during the second stage of the process.

NOTATION

α_{cr}	is the heat transfer coefficient during the constant-rate stage of desiccation;
W_{cr}	is the critical moisture content in the material;
n	is the power exponent, dependent on the properties of the dried material;
α_W	is the heat transfer coefficient during the slowing-down stage of desiccation;
T_m/T_w	is the modified Gukhman number, accounting for the assimilability of the dried material;
W/W_{cr}	is the ratio of mean moisture content in the material at any instant of time to the mean critical moisture content;
T_r/T_m	is the ratio of radiator temperature T_r to temperature of the medium inside the desiccator T_m;
ρ_0	is the density of dry material;
R_V	is the ratio of material volume to material surface;
r	is the specific heat of evaporation;
Rb	is the Rebinder number;
N	is the desiccation rate during the first stage;
T_m	is the temperature of the medium;
T_w	is the wet-bulb temperature;
$\varkappa$	is the relative desiccation factor;
$\bar{u}_{rc}$	is the referred critical moisture content;
u_e	is the equilibrium moisture content;
T_{sur}	is the surface temperature of the material;
Nu	is the Nusselt number during the slowing-down stage of desiccation;
Nu_{cr}	is the Nusselt number during the constant-rate stage of desiccation;
l	is the characteristic dimension;
λ_A	is the thermal conductivity of air;
c_M, c_0	are the specific heat of moist and of dry material, respectively;
α_V	is the volume coefficient of heat transfer;
V_d	is the volume of desiccator chamber;
F	is the surface area of particles in suspension in the desiccator chamber;
$N^* = d\bar{u}/d\tau/N$	is the dimensionless desiccation rate.

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